Name ______________________ Class ____________ Date ________

11.4 Solving Linear Systems by Multiplying First

Resource Locker

Essential Question: How can you solve a system of linear equations by using multiplication and elimination?

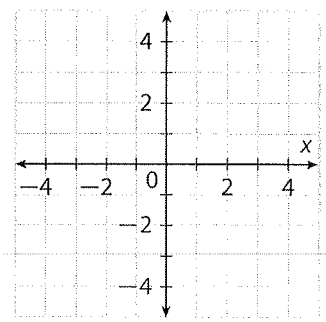
Explore 1 Understanding Linear Systems and Multiplication

A system of linear equations in which one of the like terms in each equation has either the same or opposite coefficients can that be readily solved by elimination.

How do you solve the system if neither of the pairs of like terms in the equations have the same or opposite coefficients?

(A) Graph and label the following system of equations.

$$\begin{cases} 2x - y = 1 \\ 4x + 4y = 8 \end{cases}$$



(B) The solution to the system is ____________.

(C) When both sides of an equation are multiplied by the same value, the equation [is/is not] still true.

(D) Multiply both sides of the first equation by 2.

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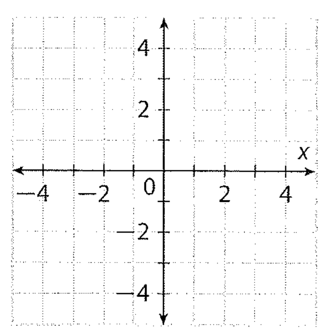
(E) Write the resulting system of equations.

$$\begin{cases} \boxed{} \\ 4x + 4y = 8 \end{cases}$$

(F) Graph and label the new system of equations.

Solution: ____________



(G) Can the new system of equations be solved using elimination now that $4x$ appears in each equation?

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Reflect

1. **Discussion** How are the graphs of $2x - y = 1$ and $4x - 2y = 2$ related?

2. **Discussion** How are the equations $2x - y = 1$ and $4x - 2y = 2$ related?

Explore 2 Proving the Elimination Method with Multiplication

The previous example illustrated that rewriting a system of equations by multiplying a constant term by one of the equations does not change the solutions for the system. What happens if a new system of equations is written by adding this new equation to the untouched equation from the original system?

(A) Original System → New System

$$\begin{cases} 2x - y = 1 \\ 4x + 4y = 8 \end{cases} \rightarrow \begin{cases} 4x - 2y = 2 \\ 4x + 4y = 8 \end{cases}$$

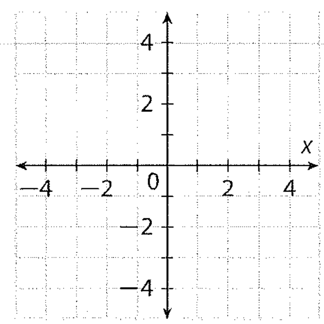
Add the equations in the new system.

$$\begin{array}{r} 4x - 2y = 2 \\ \underline{4x + 4y = 8} \end{array}$$

(B) Write a new system of equations using this new equation.

$$\begin{cases} 2x - y = 1 \\ \end{cases}$$

(C) Graph and label the equations from this new system of equations.



(D) Is the solution to this new system of equations the same as the solution to the original system of equations? Explain.

(E) If the original system is $Ax + By = C$ and $Dx + Ey = F$, where A, B, C, D, E, and F are constants, then multiply the second equation by a nonzero constant k to get $kDx + kEy = kF$. Add this new equation to $Ax + By = C$.

$$\begin{array}{rrcrcr} & Ax & + & By & = & C \\ + & kDx & + & kEy & = & kF \\ \hline & \square & + & \square & = & \square \end{array}$$

(F) So, the original system is $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$, and the new system is

$$\begin{cases} Ax + By = C \\ \underline{\qquad\qquad\qquad} . \end{cases}$$

(G) Let (x_1, y_1) be the solution to the original system. Fill in the missing parts of the following proof to show that (x_1, y_1) is also the solution to the new system.

(H) $Ax_1 + By_1 = \boxed{}$ Given.

(I) $Dx_1 + Ey_1 = \boxed{}$ Given.

(J) $\boxed{}(Dx_1 + Ey_1) = kF$ ______ Property of Equality

(K) $kDx_1 + kEy_1 = kF$ ______

(L) $\boxed{} + kDx_1 + kEy_1 = C + kF$ ______ Property of Equality

(M) $Ax_1 + \boxed{} + kDx_1 + kEy_1 = C + kF$ Substitute $Ax_1 + \boxed{}$ for $\boxed{}$ on the left.

(N) $Ax_1 + \boxed{} + By_1 + kEy_1 = C + kF$ ______ Property of Addition

(O) $(Ax_1 + kDx_1) + (By_1 + kEy_1) = C + kF$ ______ Property of Addition

(P) $(A + kD)x_1 + \left(\boxed{}\right)y_1 = C + kF$ ______

(Q) Therefore, (x_1, y_1) is the solution to the new system.

Reflect

3. **Discussion** Is a proof required using subtraction? What about division?

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Explain 1 Solving Linear Systems by Multiplying First

In some systems of linear equations, neither variable can be eliminated by adding or subtracting the equations directly. In these systems, you need to multiply one or both equations by a constant so that adding or subtracting the equations will eliminate one or more of the variables.

Steps for Solving a System of Equations by Multiplying First
1. Decide which variable to eliminate.
2. Multiply one or both equations by a constant so that adding or subtracting the equations will eliminate the variable.
3. Solve the system using the elimination method.

Example 1 **Solve each system of equations by multiplying. Check the answers by graphing the systems of equations.**

(A) $\begin{cases} 3x + 8y = 7 \\ 2x - 2y = -10 \end{cases}$

Multiply the second equation by 4.

$4(2x - 2y = -10) \Rightarrow 8x - 8y = -40$

Add the result to the first equation.

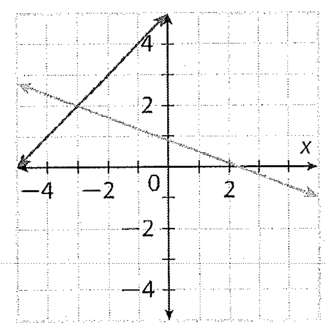
$$\begin{array}{rl} 3x + 8y = & 7 \\ +\ 8x - 8y = & -40 \\ \hline 11x \quad\quad = & -33 \end{array}$$

Solve for x.

$11x = -33$

$x = -3$

Substitute -3 for x in one of the original equations, and solve for y.



$$\begin{aligned} 3x + 8y &= 7 \\ 3(-3) + 8y &= 7 \\ -9 + 8y &= 7 \\ 8y &= 16 \\ y &= 2 \end{aligned}$$

The solution to the system is $(-3, 2)$.

(B) $\begin{cases} -3x + 2y = 4 \\ 4x - 13y = 5 \end{cases}$

Multiply the first equation by ____ and multiply the second equation by ____ so the x terms in the system have coefficients of -12 and 12 respectively.

$$\begin{array}{ll} \square(-3x + 2y = 4) & \quad -12x + \square y = \square \\ & \Rightarrow \\ \square(4x - 13y = 5) & \quad 12x - \square y = \square \end{array}$$

Add the resulting equations.

$$\begin{array}{rl} -12x + \square y = & \square \\ +12x - \square y = & \square \\ \hline \square y = & \square \end{array}$$

Solve for y.

$\square y = \square$

$y = \square$

Solve the first equation for x when $y = \boxed{}$.

$$-3x + 2y = 4$$

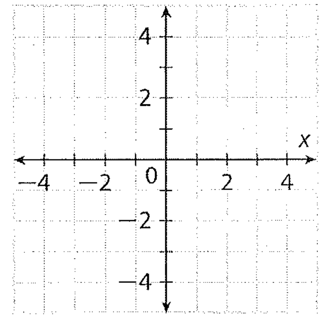
$$-3x + 2\left(\boxed{}\right) = 4$$

$$-3x + \boxed{} = 4$$

$$-3x = \boxed{}$$

$$x = \boxed{}$$

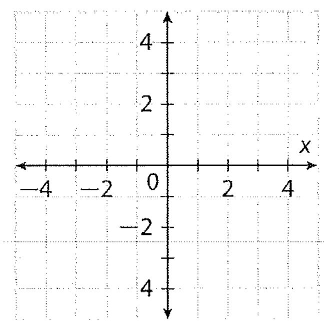
The solution to the system is $\boxed{}$.



Your Turn

Solve each system of equations by multiplying. Check the answers by graphing the systems of equations.

4. $\begin{cases} -3x + 4y = 12 \\ 2x + y = -8 \end{cases}$



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5. $\begin{cases} 2x + 3y = -1 \\ 5x - 2y = -12 \end{cases}$ $\begin{matrix})2 \rightarrow 4x + \cancel{6y} = -2 \\ )3 \rightarrow 15x - \cancel{6y} = -36 \end{matrix}$

$19x = -38$

$x = -2$

$2(-2) + 3y = -1$

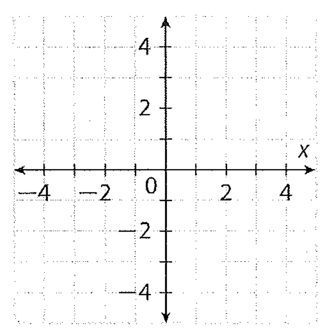
$-4 + 3y = -1$

$+4 \quad\quad +4$

$3y = 3$

$y = 1$

$(-2, 1)$



Explain 2 Solving Linear System Models by Multiplying First

You can solve a linear system of equations that models a real-world example by multiplying first.

Example 2 **Solve each problem by multiplying first.**

(A) Jessica spent \$16.30 to buy 16 flowers. The bouquet contained daisies, which cost \$1.75 each, and tulips, which cost \$0.85 each. The system of equations $\begin{cases} d + t = 16 \\ 1.75d + 0.85t = 16.30 \end{cases}$ models this situation, where d is the number of daisies and t is the number of tulips. How many of each type of flower did Jessica buy?

Multiply the first equation by -0.85 to eliminate t from each equation. Then, add the equations.

$$\begin{cases} -0.85(d + t) = -0.85(16) \\ 1.75d + 0.85t = 16.30 \end{cases} \Rightarrow \begin{aligned} -0.85d - 0.85t &= -13.60 \\ +1.75d + 0.85t &= 16.30 \\ \hline 0.9d &= 2.70 \\ d &= 3 \end{aligned}$$

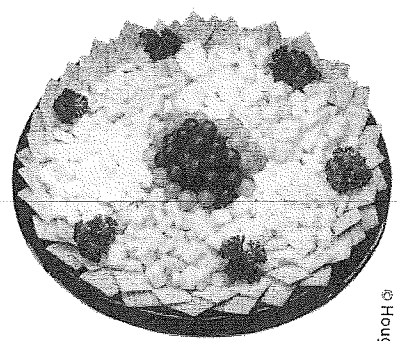
Find t.

$$\begin{aligned} d + t &= 16 \\ 3 + t &= 16 \\ t &= 13 \end{aligned}$$

The solution is (3, 13).

Jessica bought 3 daisies and 13 tulips.

(B) The Tran family is bringing 15 packages of cheese to a group picnic. Cheese slices cost \$2.50 per package. Cheese cubes cost \$1.75 per package. The Tran family spent a total of \$30 on cheese. The system of equations $\begin{cases} s + c = 15 \\ 2.50s + 1.75c = 30 \end{cases}$ represents this situation, where s is the number of packages of cheese slices and c is the number of packages of cheese cubes. How many packages of each type of cheese did the Tran family buy?



Multiply the first equation by a constant so that c can be eliminated from both equations, and then subtract the equations.

$$\begin{cases} \boxed{}(s + c) = \boxed{}(15) \\ 2.50s + 1.75c = 30 \end{cases} \Rightarrow \begin{aligned} \boxed{}\,s + \boxed{}\,c &= \boxed{} \\ -(2.50s + 1.75c\,) &= -30 \\ \hline \boxed{}\,s &= \boxed{} \\ s &= \boxed{} \end{aligned}$$

Find c.

$$\begin{aligned} s + c &= 15 \\ \boxed{} + c &= 15 \\ c &= \boxed{} \end{aligned}$$

The solution is $\boxed{}$.

The Tran family bought ______ packages of sliced cheese and ______ packages of cheese cubes.

Your Turn

6. Jacob's family bought 4 adult tickets and 2 student tickets to the school play for \$64. Tatianna's family bought 3 adult tickets and 3 students tickets for \$60. The system of equations $\begin{cases} 4a + 2s = 64 \\ 3a + 3s = 60 \end{cases}$ models this situation, where a is the cost of an adult ticket and s is the cost of a student ticket. How much does each type of ticket cost?

Elaborate

7. When would you solve a system of linear equations by multiplying?

8. How can you use multiplication to solve a system of linear equations if none of the coefficients are multiples or factors of any of the other coefficients?

9. **Essential Question Check-In** How do you solve a system of equations by multiplying?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

For each linear equation,

a. **find the product of 3 and the linear equation;**

b. **solve both equations for y.**

1. $2y - 4x = 8$

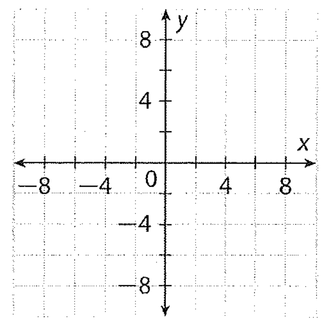
2. $-5y + 7x = 12$

3. $4x + 7y = 18$

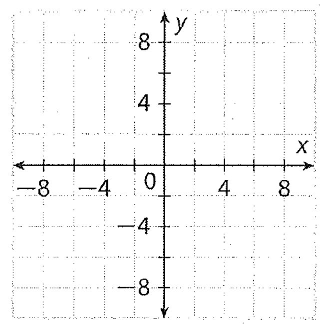
4. $x - 2y = 13$

For each linear system, multiply the first equation by 2 and add the new equation to the second equation. Then, graph this new equation along with both of the original equations.

5. $\begin{cases} 2x + 4y = 24 \\ -12x + 8y = -16 \end{cases}$

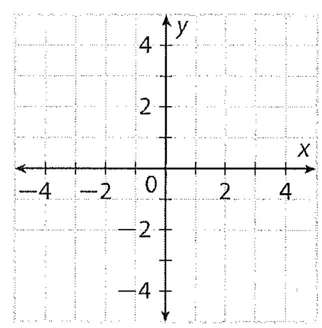


6. $\begin{cases} 2x + 2y = 16 \\ -15x + 3y = -12 \end{cases}$

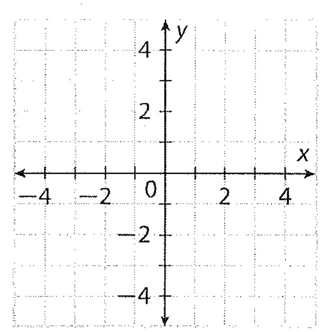


Solve each system of linear equations by multiplying. Verify each answer by graphing the system of equations.

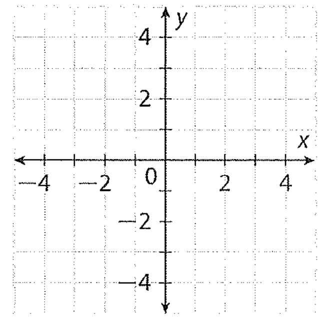
7. $\begin{cases} 5x - 2y = 11 \\ 3x + 5y = 19 \end{cases}$



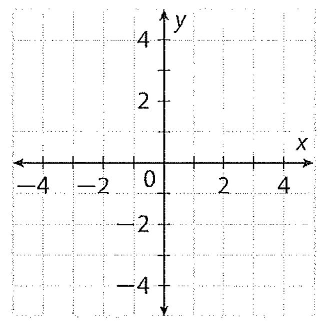
8. $\begin{cases} -2x + 2y = 2 \\ -4x + 7y = 16 \end{cases}$



9. $\begin{cases} 3x + 4y = 13 \\ 2x - 2y = -10 \end{cases}$



10. $\begin{cases} x - 4y = -1 \\ 5x + 2y = 17 \end{cases}$



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Solve each system of linear equations using multiplication.

11. $\begin{cases} (-3x + 2y = 4)3 \\ (5x - 3y = 1)2 \end{cases}$ $\longrightarrow$ $-9x + \cancel{6y} = 12$ $\longrightarrow$ $10x - \cancel{6y} = 2$

$x = 14$

$(14, 23)$

$-3(14) + 2y = 4$

$-42 + 2y = 4$

$+42 \quad +42$

$2y = 46$

$y = 23$

12. $\begin{cases} 3x + 3y = 12 \\ 6x + 11y = 14 \end{cases}$

Solve each problem by multiplying first.

13. The sum of two angles is 180°. The difference between twice the larger angle and three times the smaller angle is 150°. The system of equations $\begin{cases} x + y = 180 \\ 2x - 3y = 150 \end{cases}$ models this situation, where x is the measure of the larger angle and y is the measure of the smaller angle. What is the measure of each angle?

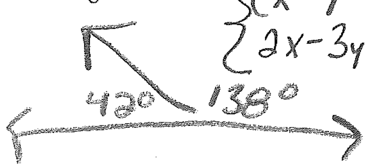
$\begin{cases} (x + y = 180)-2 \\ 2x - 3y = 150 \end{cases}$

$-\cancel{2x} - 2y = -360$

$\cancel{2x} - 3y = 150$

$-5y = -210$

$y = 42$



14. The perimeter of a rectangular swimming pool is 126 feet. The difference between the length and the width is 39 feet. The system of equations $\begin{cases} 2x + 2y = 126 \\ x - y = 39 \end{cases}$ models this situation, where x is the length of the pool and y is the width of the pool. Find the dimensions of the swimming pool.

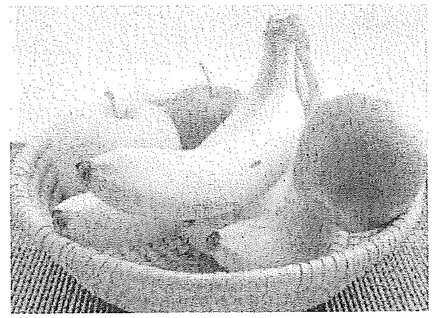
15. Jamian bought a total of 40 bagels and donuts for a morning meeting. He paid a total of \$33.50. Each donut cost \$0.65 and each bagel cost \$1.15. The system of equations $\begin{cases} b + d = 40 \\ 1.15b + 0.65d = 33.50 \end{cases}$ models this situation, where b is the number of bagels and d is the number of donuts. How many of each did Jamian buy?

16. A clothing store is having a sale on shirts and jeans. 4 shirts and 2 pairs of jeans cost \$64. 3 shirts and 3 pairs of jeans cost \$72. The system of equations $\begin{cases} 4s + 2j = 64 \\ 3s + 3j = 72 \end{cases}$ models this situation, where s is the cost of a shirt and j is the cost of a pair of jeans. How much does one shirt and one pair of jeans cost?

17. Jayce bought 5 bath towels and returned 2 hand towels. His sister Jayna bought 3 bath towels and returned 4 hand towels. Jayce paid a total of \$124 and Jayna paid a total of \$24. The system of equations $\begin{cases} 5b - 2h = 124 \\ 3b - 4h = 24 \end{cases}$ models this situation, where b is the price of a bath towel and h is the price of a hand towel. How much does each kind of towel cost?

18. Apples cost \$0.95 per pound and bananas cost \$1.10 per pound. Leah bought a total of 8 pounds of apples and bananas for \$8.05.

The system of equations $\begin{cases} a + b = 8 \\ 0.95a + 1.10b = 8.05 \end{cases}$ models this situation, where a is the number of pounds of apples and b is the number of pounds of bananas. How many pounds of each did Leah buy?



19. Which of the following are possible ways to eliminate a variable by multiplying first? $\begin{cases} -x + 2y = 3 \\ 4x - 5y = -3 \end{cases}$

a. Multiply the first equation by 4.

b. Multiply the first equation by 5 and the second equation by 2.

c. Multiply the first equation by 4 and the second equation by 2.

d. Multiply the first equation by 5 and the second equation by 4.

e. Multiply the first equation by 2 and the second equation by 5.

f. Multiply the second equation by 4.

20. **Explain the Error** A linear system has two equations $Ax + By = C$ and $Dx + Ey = F$. A student begins to solve the equation as shown. What is the error?

$$\begin{aligned} Ax + By &= C \\ + k(Dx + Ey) &= F \\ \hline (A + kD)x + (B + kE)y &= C + F \end{aligned}$$

21. **Critical Thinking** Suppose you want to eliminate y in this system: $\begin{cases} 2x + 11y = -3 \\ 3x + 4y = 8 \end{cases}$

By what numbers would you need to multiply the two equations in order to eliminate y? Why might you choose to eliminate x instead?

H.O.T. Focus on Higher Order Thinking

22. **Justify Reasoning** Solve the following system of equations by multiplying.

$\begin{cases} x + 3y = -14 \\ 2x + y = -3 \end{cases}$ Would it be easier to solve the system by using substitution? Explain your reasoning.