Name ______________________ Class 2 ____________ Date ________

11.4 AA Similarity of Triangles

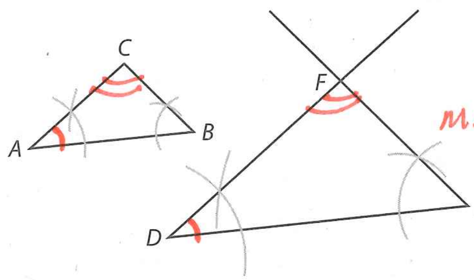
Essential Question: How can you show that two triangles are similar?

Resource Locker

Explore Exploring Angle-Angle Similarity for Triangles

Two triangles are similar when their corresponding sides are proportional and their corresponding angles are congruent. There are several shortcuts for proving triangles are similar.

(A) Draw a triangle and label it $\triangle ABC$. Elsewhere on your page, draw a segment longer than $\overline{AB}$ and label the endpoints D and E.



m∠B ≅ m∠E
△ AA~

(B) Copy $\angle CAB$ and $\angle ABC$ to points D and E, respectively. Extend the rays of your copied angles, if necessary, and label their intersection point F. You have constructed $\triangle DEF$.

(C) You constructed angles D and E to be congruent to angles A and B, respectively. Therefore,

angles C and F must also be __________ because of the __________ Theorem.

(D) Check the proportionality of the corresponding sides.

$\frac{AB}{DE} = \frac{\square}{\square} = \square$ $\quad \frac{AC}{DF} = \frac{\square}{\square} = \square$ $\quad \frac{BC}{EF} = \frac{\square}{\square} = \square$

Since the ratios are ________ the sides of the triangles are __________.

Reflect

1. **Discussion** Compare your results with your classmates. What conjecture can you make about two triangles that have two corresponding congruent angles?

____

____

Explain 1 Proving Angle-Angle Triangle Similarity

The Explore suggests the following theorem for determining whether two triangles are similar.

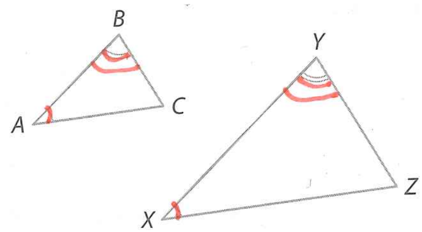
Angle-Angle (AA) Triangle Similarity Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.

Example 1 **Prove the Angle-Angle Triangle Similarity Theorem.**

Given: $\angle A \cong \angle X$ and $\angle B \cong \angle Y$

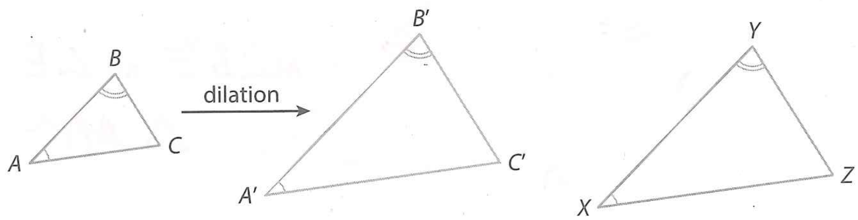
Prove: $\triangle ABC \sim \triangle XYZ$



$m\angle C \cong m\angle Z$

By $\triangle$ AA $\sim$

① Apply a dilation to $\triangle ABC$ with scale factor $k = \frac{XY}{AB}$. Let the image of $\triangle ABC$ be $\triangle A'B'C$.



$\triangle A'B'C$ is similar to $\triangle ABC$, and $\angle A' \cong$ ______ and $\angle B' \cong$ ______

because ______________________________.

Also, $A'B' = k \cdot AB =$ ______________.

② It is given that $\angle A \cong \angle X$ and $\angle B \cong \angle Y$

By the Transitive Property of Congruence, $\angle A' \cong$ ______ and $\angle B' \cong$ ______.

So, $\triangle A'B'C' \cong \triangle XYZ$ by ______________________________.

This means there is a sequence of rigid motions that maps $\triangle A'B'C'$ to $\triangle YYZ$.

The dilation followed by this sequence of rigid motions shows that there is a sequence of similarity transformations that maps $\triangle ABC$ to $\triangle XYZ$. Therefore, $\triangle ABC \sim \triangle XYZ$.

Reflect

2. **Discussion** Compare and contrast the AA Similarity Theorem with the ASA Congruence Theorem.

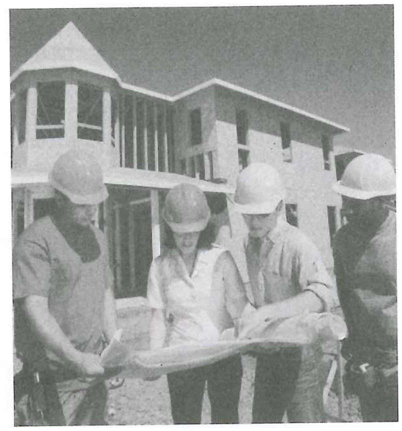
3. In $\triangle JKL$, $m\angle J = 40°$ and $m\angle K = 55°$. In $\triangle MNP$, $m\angle M = 40°$ and $m\angle P = 85°$. A student concludes that the triangles are not similar. Do you agree or disagree? Why?

____

____

Explain 2 Applying Angle-Angle Similarity

Architects and contractors use the properties of similar figures to find any unknown dimensions, like the proper height of a triangular roof. They can use a bevel angle tool to check that the angles of construction are congruent to the angles in their plans.

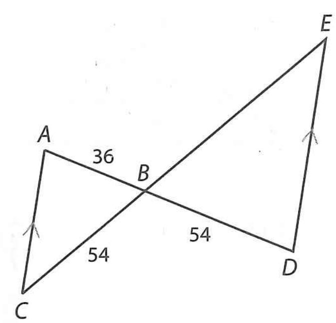


Example 2 **Find the indicated length, if possible.**

Ⓐ BE

First, determine whether $\triangle ABC \sim \triangle DBE$.

By the Alternate Interior Angles Theorem, $\angle A \cong \angle D$ and $\angle C \cong \angle E$, so $\triangle ABC \sim \triangle DBE$ by the AA Triangle Similarity Theorem.



Find BE by solving a proportion.

$$\frac{BD}{BA} = \frac{BE}{BC}$$

$$\frac{54}{36} = \frac{BE}{54}$$

$$\frac{54}{36} \cdot 54 = \frac{BE}{54} \cdot 54$$

$$BE = 81$$

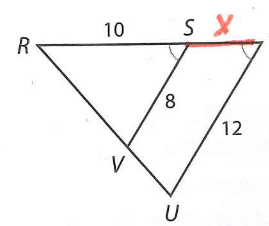
Ⓑ RT

Check whether $\triangle RSV \sim \triangle RTU$:

It is given in the diagram that $\angle$ $\boxed{S}$ $\cong \angle$ $\boxed{T}$. $\angle R$ is shared by both triangles,

so $\angle R \cong \angle R$ by the Reflexive Property of Congruence.

So, by the ______________________, $\triangle RST \sim \triangle RTU$.



Find RT by solving a proportion.

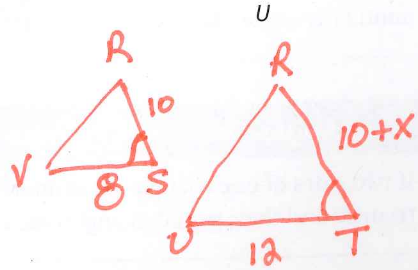
$$\frac{RT}{RS} = \frac{TU}{SV}$$

$$\frac{\cancel{RT}\ X}{\boxed{10}} = \frac{\boxed{12}}{\boxed{8}}$$

Proportion

$$\frac{8X}{8} = \frac{120}{8}$$

$$X = \frac{60}{4} = 15$$



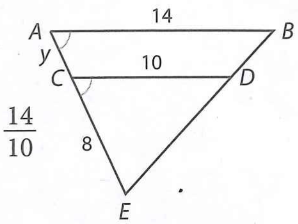
$RT = \boxed{15}$

$X = 15$

Reflect

4. In Example 2*A*, is there another way you can set up the proportion to solve for *BE*?

5. **Discussion** When asked to solve for *y*, a student sets up the proportion as shown. Explain why the proportion is wrong. How should you adjust the proportion so that it will give the correct result?

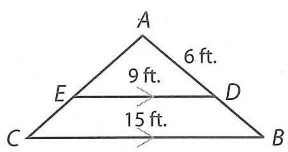


$$\frac{y}{8} = \frac{14}{10}$$

Your Turn

Pg 614

6. A builder was given a design plan for a triangular roof as shown. Explain how he knows that $\triangle AED \sim \triangle ACB$. Then find *AB*.

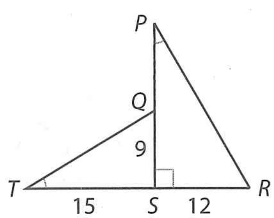


$\frac{9}{15} = \frac{6}{AB}$

$9(AB) = 90$

$AB = 10$ ft

7. Find *PQ*, if possible.



$PQ = 11$

$PS = 20$

$\frac{9}{12} = \frac{15}{9+PQ}$

$9(9+PQ) = 12 \cdot 15$

$81 + 9pq = 180$

$9pq = 99$

$pq = 11$

Explain 3 Applying SSS and SAS Triangle Similarity

In addition to Angle-Angle Triangle Similarity, there are two additional shortcuts for proving two triangles are similar.

Side-Side-Side (SSS) Triangle Similarity Theorem

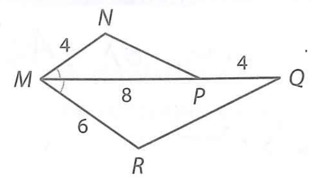
If the three sides of one triangle are proportional to the corresponding sides of another triangle, then the triangles are similar.

Side-Angle-Side (SAS) Triangle Similarity Theorem

If two sides of one triangle are proportional to the corresponding sides of another triangle and their included angles are congruent, then the triangles are similar.

Example 3 Determine whether the given triangles are similar. Justify your answer.

(A)



You are given two pairs of corresponding side lengths and one pair of congruent corresponding angles, so try using SAS.

Check that the ratios of corresponding sides are equal.

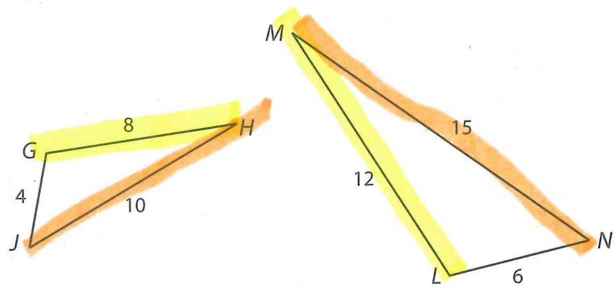
$\frac{MN}{MR} = \frac{4}{6} = \frac{2}{3}$ $\qquad \frac{MP}{MQ} = \frac{8}{8+4} = \frac{8}{12} = \frac{2}{3}$

Check that the included angles are congruent: $\angle NMP \cong \angle QMR$ is given in the diagram.

Therefore $\triangle NMP \sim \triangle RMQ$ by the SAS Triangle Similarity Theorem.

pg 615

(B)



You are given 3 pairs of corresponding side lengths and 3 congruent corresponding angles, so try using SSS ~.

Check that the ratios of corresponding sides are equal.

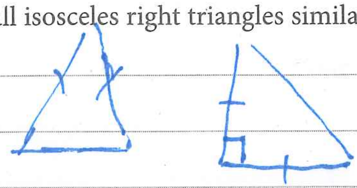
$\frac{LM}{GH} = \frac{12}{8} = \frac{3}{2} \qquad \frac{MN}{HJ} = \frac{15}{10} = \frac{3}{2} \qquad \frac{LN}{GJ} = \frac{6}{4} = \frac{3}{2}$

Therefore $\triangle$ JHG $\sim \triangle$ NML by $\triangle$ SSS ~.

Since you are given all three pairs of sides, you don't need to check for congruent angles.

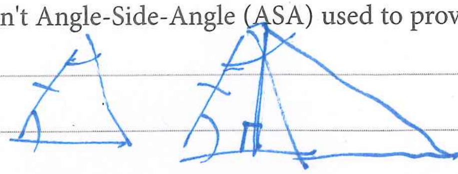
Reflect

8. Are all isosceles right triangles similar? Explain why or why not.



yes, SAS

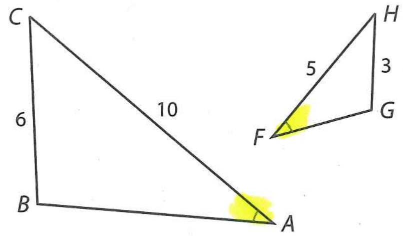
9. Why isn't Angle-Side-Angle (ASA) used to prove two triangles similar?



Your Turn

If possible, determine whether the given triangles are similar. Justify your answer.

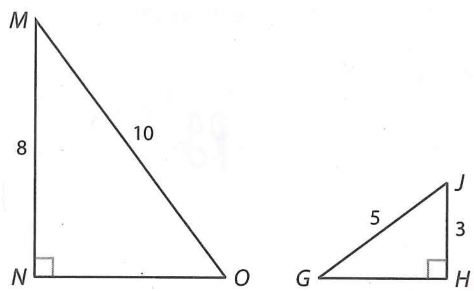
10.



AA~, SSS~, or SAS~

No ASS~

11.



Elaborate

12. Is triangle similarity transitive? If you know $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHJ$, is $\triangle ABC \sim \triangle GHJ$? Explain.

13. The AA Similarity Theorem applies to triangles. Is there an AAA Similarity Theorem for quadrilaterals? Use your geometry software to test your conjecture or create a counterexample.

14. **Essential Question Check-In** How can you prove triangles are similar?