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BENCHMARK 4
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D. Properties of Exponents (pp. 70–73)

A **power** represents repeated multiplication. The following examples illustrate several properties that you can use to simplify expressions with powers.

1. Product of Powers Property

Vocabulary **Product of Powers Property** To multiply powers having the same base, add the exponents.

EXAMPLE **Simplify the expression.**

a. $4^2 \cdot 4^8$ **b.** $3 \cdot 3^7 \cdot 3^6$ **c.** $(-12)(-12)^4$ **d.** $x^2 \cdot x^{10} \cdot x^5$

If a is a real number and m and n are positive integers, $a^m \cdot a^n = a^{m+n}$.

Solution:

a. $4^2 \cdot 4^8 = 4^{2+8} = 4^{10}$

b. $3 \cdot 3^7 \cdot 3^6 = 3^1 \cdot 3^7 \cdot 3^6 = 3^{1+7+6} = 3^{14}$

c. $(-12)(-12)^4 = (-12)^1 \cdot (-12)^4 = (-12)^{1+4} = (-12)^5$

d. $x^2 \cdot x^{10} \cdot x^5 = x^{2+10+5} = x^{17}$

PRACTICE **Simplify the expression.**

1. $8^7 \cdot 8^2$ **2.** $5^3 \cdot 5 \cdot 5^9$ **3.** $(-6)^7(-6)^4(-6)$

4. $d^{10} \cdot d^7$ **5.** $r^5 \cdot r^7 \cdot r^8$ **6.** $k^2 \cdot k^3 \cdot k$

2. Power of a Power Property

Vocabulary **Power of a Power Property** To find a power of a power, multiply exponents.

EXAMPLE **Simplify the expression.**

a. $(3^2)^4$ **b.** $[(-4)^9]^2$ **c.** $(x^3)^3$ **d.** $[(y-3)^5]^4$

If a is a real number and m and n are positive integers, $(a^m)^n = a^{mn}$.

Solution:

a. $(3^2)^4 = 3^{2 \cdot 4} = 3^8$

b. $[(-4)^9]^2 = (-4)^{9 \cdot 2} = (-4)^{18}$

c. $(x^3)^3 = x^{3 \cdot 3} = x^9$

d. $[(y-3)^5]^4 = (y-3)^{5 \cdot 4} = (y-3)^{20}$

PRACTICE **Simplify the expression.**

7. $(10^3)^6$ **8.** $[(-6)^2]^8$ **9.** $(t^5)^3$

10. $(b^4)^4$ **11.** $[(p+5)^7]^2$ **12.** $[(h-1)^9]^5$

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3. Power of a Product Property

Vocabulary

Power of a Product Property To find a power of a product, find the power of each factor and multiply.

EXAMPLE

Evaluate the numerical power when simplifying powers with both numerical and variable bases.

Simplify the expression.

a. $(9 \cdot 7)^3$ **b.** $(5xy)^2$ **c.** $(-3z)^4$ **d.** $-(3z)^4$

Solution:

a. $(9 \cdot 7)^3 = 9^3 \cdot 7^3$

b. $(5xy)^2 = (5 \cdot x \cdot y)^2 = 5^2 \cdot x^2 \cdot y^2 = 25x^2y^2$

c. $(-3z)^4 = (-3 \cdot z)^4 = (-3)^4 \cdot z^4 = 81z^4$

d. $-(3z)^4 = -(3 \cdot z)^4 = -(3^4 \cdot z^4) = -81z^4$

PRACTICE

If a and b are real numbers and m is a positive integer, $(ab)^m = a^m b^m$.

Simplify the expression.

13. $(5 \cdot 4)^6$ **14.** $(3gh)^3$ **15.** $(6cd)^2$

16. $(-2p)^4$ **17.** $-(5t)^3$ **18.** $-(-8a)^2$

4. Quotient of Powers Property

Vocabulary

Quotient of Powers Property To divide powers having the same base, subtract exponents.

EXAMPLE

If a is a nonzero real number and m and n are positive integers such that $m > n$, $\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$.

Simplify the expression.

a. $\frac{9^7}{9^3}$ **b.** $\frac{(-6)^{10}}{(-6)^8}$ **c.** $\frac{3^9 \cdot 3^5}{3^4}$ **d.** $\frac{1}{x^5} \cdot x^{12}$

Solution:

a. $\frac{9^7}{9^3} = 9^{7-3} = 9^4$

b. $\frac{(-6)^{10}}{(-6)^8} = (-6)^{10-8} = (-6)^2$

c. $\frac{3^9 \cdot 3^5}{3^4} = \frac{3^{14}}{3^4} = 3^{14-4} = 3^{10}$

d. $\frac{1}{x^5} \cdot x^{12} = \frac{x^{12}}{x^5} = x^{12-5} = x^7$

PRACTICE

Simplify the expression.

19. $\frac{18^{23}}{18^{17}}$ **20.** $\frac{2^{35}}{2^3}$ **21.** $\frac{(-25)^3}{(-25)}$

22. $\frac{4^6 \cdot 4^9}{4^3}$ **23.** $\frac{1}{n^8} \cdot n^{12}$ **24.** $w^6 \cdot \frac{1}{w^4}$

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5. Power of a Quotient Property

Vocabulary

Power of a Quotient Property To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

EXAMPLE **Simplify the expression.**

If a and b are real numbers and m is a positive integer, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$.

a. $\left(\frac{a}{b}\right)^7$ **b.** $\left(-\frac{2}{x}\right)^3$

Solution:

a. $\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$

b. $\left(-\frac{2}{x}\right)^3 = \left(\frac{-2}{x}\right)^3 = \frac{(-2)^3}{x^3} = -\frac{8}{x^3}$

PRACTICE **Simplify the expression.**

25. $\left(\frac{2}{5}\right)^3$ **26.** $\left(-\frac{5}{8}\right)^2$ **27.** $\left(\frac{1}{p}\right)^8$

28. $\left(\frac{r}{s}\right)^6$ **29.** $\left(-\frac{u}{v}\right)^9$ **30.** $\left(-\frac{2}{b}\right)^5$

6. Zero and Negative Exponents

Vocabulary

Zero power a to the zero power is 1.
Negative exponents a^{-n} is the reciprocal of a^n, a^n is the reciprocal of a^{-n}.

EXAMPLE **Simplify the expression.**

a. 5^{-3} **b.** $(-12)^0$ **c.** $\left(\frac{1}{2}\right)^{-5}$ **d.** 0^{-4}

If a is a real number ($a \neq 0$) and n is an integer, $a^0 = 1$, $a^{-n} = \frac{1}{a^n}$, and $a^n = \frac{1}{a^{-n}}$.

Solution:

a. $5^{-3} = \frac{1}{5^3}$
$= \frac{1}{125}$

b. $(-12)^0 = 1$

c. $\left(\frac{1}{2}\right)^{-5} = \frac{1}{\left(\frac{1}{2}\right)^5}$
$= \frac{1}{\frac{1}{32}}$
$= 32$

d. $0^{-4} = \frac{1}{0^4}$ (crossed out)
Division by 0 is undefined.

PRACTICE **Simplify the expression.**

31. 11^{-2} **32.** $\left(\frac{5}{8}\right)^0$ **33.** $\left(\frac{1}{4}\right)^{-3}$

34. $(-3)^{-4}$ **35.** $\frac{1}{6^{-2}}$ **36.** $(-25)^0$

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Quiz

Simplify the expression.

1. $7^4 \cdot 7 \cdot 7^5$	**2.** $(-8)^3(-8)^9(-8)$	**3.** $y^{12} \cdot y^8 \cdot y^9$
4. $(6^5)^3$	**5.** $[(-4)^3]^7$	**6.** $(s^6)^2$
7. $[(a+11)^8]^4$	**8.** $(8 \cdot 2)^{13}$	**9.** $(-5jk)^7$
10. $\frac{3^9 \cdot 3^6}{3^2}$	**11.** $\frac{1}{v^8} \cdot v^{17}$	**12.** $\left(-\frac{3}{4}\right)^3$
13. $\left(\frac{4}{h}\right)^2$	**14.** $\left(-\frac{p}{q}\right)^{15}$	**15.** $\left(\frac{2}{7}\right)^0$
16. $(-2)^{-5}$	**17.** $\frac{1}{7^{-2}}$	**18.** $(-13)^0$