

9.3 Properties of Rectangles, Rhombuses, and Squares

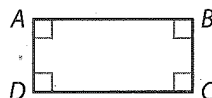


Resource Locker

Essential Question: What are the properties of rectangles, rhombuses, and squares?

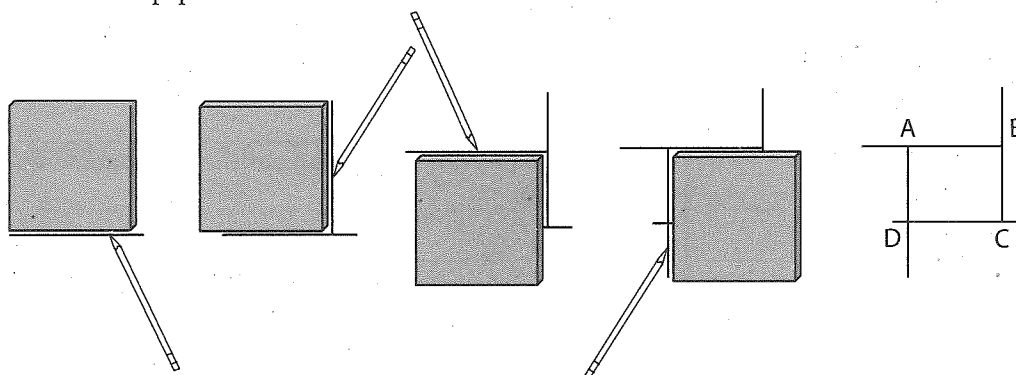
Explore Exploring Sides, Angles, and Diagonals of a Rectangle

A **rectangle** is a quadrilateral with four right angles. The figure shows rectangle $ABCD$.



Investigate properties of rectangles.

- A** Use a tile or pattern block and the following method to draw three different rectangles on a separate sheet of paper.



- B** Use a ruler to measure the sides and diagonals of each rectangle. Keep track of the measurements and compare your results to other students.

Reflect

1. Why does this method produce a rectangle? What must you assume about the tile?

2. **Discussion** Is every rectangle also a parallelogram? Make a conjecture based upon your measurements and explain your thinking.

3. Use your measurements to make two conjectures about the diagonals of a rectangle.

Conjecture: _____

Conjecture: _____

Explain 1 Proving Diagonals of a Rectangle are Congruent

You can use the definition of a rectangle to prove the following theorems.

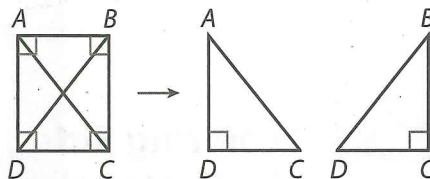
Properties of Rectangles

If a quadrilateral is a rectangle, then it is a parallelogram.
If a parallelogram is a rectangle, then its diagonals are congruent.

Example 1 Use a rectangle to prove the Properties of Rectangles Theorems.

Given: $ABCD$ is a rectangle.

Prove: $ABCD$ is a parallelogram; $\overline{AC} \cong \overline{BD}$.



Statements	Reasons
1. $ABCD$ is a rectangle.	1. Given
2. $\angle A$ and $\angle C$ are right angles.	2. Definition of
3. $\angle A \cong \angle C$	3. All right angles are congruent.
4. $\angle B$ and $\angle D$ are right angles.	4.
5. $\angle B \cong \angle D$	5.
6. $ABCD$ is a parallelogram.	6.
7. $\overline{AD} \cong \overline{CB}$	7. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
8. $\overline{DC} \cong \overline{DC}$	8.
9. $\angle D$ and $\angle C$ are right angles.	9. Definition of rectangle
10. $\angle D \cong \angle C$	10. All right angles are congruent.
11.	11.
12.	12.

Reflect

4. **Discussion** A student says you can also prove the diagonals are congruent in Example 1 by using the SSS Triangle Congruence Theorem to show that $\triangle ADC \cong \triangle BCD$. Do you agree? Explain.

Your Turn

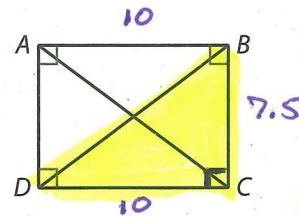
Find each measure.

5. $AD = 7.5$ cm and $DC = 10$ cm. Find DB .

6. $AB = 17$ cm and $BC = 12.75$ cm. Find DB .

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 7.5^2 + 10^2 &= (DB)^2 \\
 56.25 + 100 &= (DB)^2 \\
 \sqrt{156.25} &= (DB) \\
 12.5 &= DB
 \end{aligned}$$

$$\begin{aligned}
 a^2 + b^2 &= c^2 \\
 17^2 + 12.75^2 &= (DB)^2 \\
 289 + 162.5625 &= (DB)^2 \\
 \sqrt{451.5625} &= DB \\
 21.25 &= DB
 \end{aligned}$$

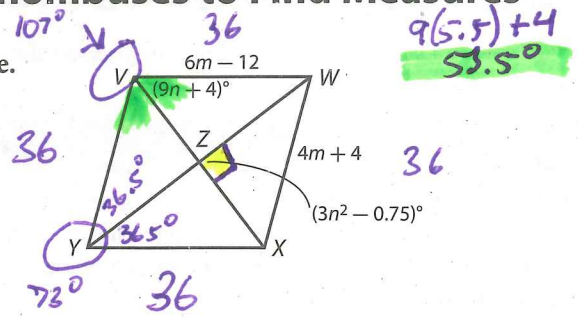


Explain 3 Using Properties of Rhombuses to Find Measures

Example 3 Use rhombus VWXY to find each measure.

$3n^2 - 0.75 = 90$
 $+ 0.75 + 0.75$
 $3n^2 = 90.75$
 $\sqrt{3n^2} = \sqrt{90.75}$
 $n^2 = 30.25$
 $n = 5.5$

$VW = WX$
 $6m - 12 = 4m + 4$
 $- 4m - 4m$
 $2m - 12 = 4$
 $+ 12 + 12$
 $2m = 16$
 $m = 8$



A Find XY.

All sides of a rhombus are congruent, so $\overline{VW} \cong \overline{WX}$ and $VW = WX$.

Substitute values for VW and WX. $6m - 12 = 4m + 4$

Solve for m . $m = 8$

Substitute the value of m to find VW. $VW = 6(8) - 12 = 36$

Because all sides of the rhombus are congruent, then $\overline{VW} \cong \overline{XY}$, and $XY = 36$.

B Find $\angle YVW$.

The diagonals of a rhombus are _____, so $\angle WZX$ is a right angle and

$m\angle WZX = \square$

Since $m\angle WZX = (3n^2 - 0.75)^\circ$, then _____

Solve for n . $3n^2 - 0.75 = 90$

$n = \square$

Substitute the value of n to find $m\angle WVZ$.

$m\angle WVZ = \square$

Since \overline{VX} bisects $\angle YVW$, then _____

Substitute 53.5° for $m\angle WVZ$. $m\angle YVW = 2(53.5^\circ) = 107^\circ$

Your Turn

Use the rhombus VWXY from Example 3 to find each measure.

10. Find $m\angle VYX$.

73°

11. Find $m\angle XYZ$.

36.5°